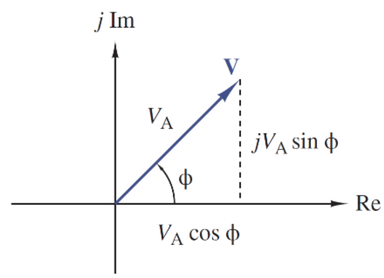
Notes on phasors in electrical engineering

Logan Thrasher Collins

- Phasors are complex representations of sinusoidal (usually cosine) functions with constant angular frequency, amplitude, and initial phase. In electrical engineering, they are used to simplify circuit analysis differential equations to algebraic equations. A cosine function and its equivalent phasor are given below.

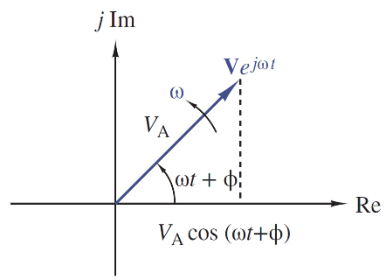
$$v_A(t) = V_A \cos(\omega t - \phi)$$

$$\mathbf{V} = V_A \angle \phi$$



- The phasor domain is constructed using the complex plane. The amplitude corresponds to the magnitude of a complex number, while the phase shift corresponds to the angle of the complex number with respect the real axis.
- Using Euler's formula, cosine functions are converted to exponential form. $V_A \angle \varphi$ is shorthand for $V_A e^{j\varphi}$.

$$\mathbf{V} = V_A e^{j\phi} = \mathrm{Re}\{\cos(\phi) + j\sin(\phi)\}$$



- When adding sinusoids that possess the same frequency, their phasor representation is a sum of the component phasors.

$$\mathbf{V} = \sum \mathbf{V}_i$$

- Using the 90° phase shift that comes from taking the derivative of a sinusoid, the following expression is obtained. As such, multiplying a phasor by jω is equivalent to differentiating the corresponding sinusoid.

$$j\omega\mathbf{V} = V_A e^{j(\phi + 90°)}$$

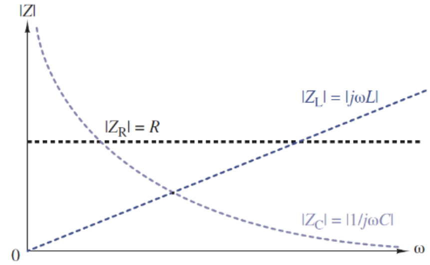
- Phasors are converted into complex numbers using their geometry. For instance, consider a phasor 5∠30°. The corresponding complex number is 5cos(30°) + 5sin(30°)j = 4.33 + 2.5j.
- Phasors can be applied to circuit analysis when the voltage or current sources take on sinusoidal waveforms.
- Kirchhoff's voltage law and current law applies to phasor voltages and phasor currents.
- Impedance is a generalization of resistance. As such, all passive circuit elements are treated as resistors (except that the given element's impedance is used rather than resistance). The impedances of resistors, capacitors, and inductors are given below. Impedance is a complex quantity with units of ohms.

$$Z_R = R, \qquad Z_C = \frac{1}{j\omega C} = -\frac{j}{\omega C}, \qquad Z_L = j\omega L$$

- An analogous version of Ohm's law is true for impedance.

$$\mathbf{V} = Z\mathbf{I}$$

- As a result of the equations for impedance, frequency influences the passive circuit elements (resistors, capacitors, and inductors) in distinct ways. The relationship between the magnitude of the impedance and the frequency are plotted at right.



- To carry out phasor analysis on a circuit, the following steps are performed.
    1. Convert sinusoidal voltage and current sources into the phasor domain and characterize passive current elements via their impedances.
    2. Use standard algebraic circuit analysis techniques (i.e. KCL, KVL, source transformations, etc.) to solve the circuit in terms of its phasor domain quantities.
    3. Transform the phasor domain responses back into their corresponding time domain responses to obtain the waveforms across each circuit element.
- Impedances possess series equivalence. The real part of Z is called the resistance and the imaginary part is called the reactance. If the imaginary part X is negative, it is called a capacitive reactance. If the imaginary part X is positive, it is called an inductive reactance.

$$Z_{eq} = \sum Z_i = R + jX$$

- Voltage division is carried out in the phasor domain using the following equation.

$$\mathbf{V}_k = Z_k \mathbf{I} = \frac{Z_k}{Z_{eq}} \mathbf{V}$$

- Impedances possess parallel equivalence and can be written in terms of the admittance Y = 1/Z. Admittance has units of siemens (S). The real part of Y is called the conductance G and the imaginary part is called the susceptance B.

$$Y_{eq} = \sum Y_i = \frac{1}{Z_{eq}} = \sum \frac{1}{Z_i} = G + jB$$

- Current division is carried out in the phasor domain using the following equation.

$$\mathbf{I}_k = Y_k \mathbf{V} = \frac{Y_k}{Y_{eq}} \mathbf{I}$$

Reference and source of images: Thomas, R. E., Rosa, A. J., & Toussaint, G. J. (2016). *The Analysis and Design of Linear Circuits* (8th ed.). Wiley.